Ultrasonic discrimination and modelling for crack-tip echoes

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Abstract

The problem of discriminating between the acoustical signatures of an open crack tip and an interface is approached by modeling the various ultrasonic signatures, and by a signal-processing method. In the modeling study, the Kirchhoff approximation (the physical optics approximation) is used to describe the far field scattered by an arbitrary object with non-negligible dimensions in comparison with the wavelength, and to express this field depending on the incident field. It is established that, under this condition, the impulse response of an open crack tip is proportional to the first-order derivative of the impulse response of the transmitter. In the signal processing study, spectral and wavelet analysis was applied to the discrimination problem. The basic idea was to discriminate signatures simultaneously in time and in frequency (scale) domain. This method was found to be an effective means of testing models for the interactions between waves and flaws. Both aspects (modeling and signal processing) were studied numerically and experimentally, and the validity of the results was tested on an industrial sample.

Keywords: Ultrasonic, crack detection, non-destructive testing, acoustical signature discrimination, spectral analysis, wavelet decomposition

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Running title: Ultrasonic crack discrimination and modeling

Introduction

Ultrasonic crack detection is a signal discrimination problem: one has to differentiate between the signatures of flawed regions of the material and those of intact regions. In the field of ultrasonic inspection methods, echoes of several kinds are known to occur, since they can be generated by interfaces, by the structure, by the body or tips of cracks, or by inclusions. Those echoes result from various acoustic processes [1], ranging, under approximations, between two extremes: pure reflection where the echoes are "copies" of the emitted signal, with some distortions due to the propagation, and pure scattering where the echoes are "copies" of the derivative of the emitted signal, again with some distortions [2]. The spectral signatures of these various processes differ significantly, and the idea naturally arose of attempting to model crack/wave interactions and to use a wavelet analysis to discriminate different ultrasonic waves phenomena.

Ultimately, our aim is to solve an inverse scattering problem (the discrimination of defaults), and first the corresponding forward problem must be solved. Different methods have been applied to the forward problem, including equation integral, hybrid finite-elements methods and the geometrical theory of diffraction. The latter method provides asymptotic approximations for diffracted fields, valid for high frequencies and large distances from the diffracting body [3, 4, 5]. Several strategies are possible allowing simple, efficient, and precise numerical modeling of the forward and the inverse problem. With the Kirchhoff approximation, also known as the physical optics approximation, which applies to non-penetrable targets and when the wavelength is smaller than the characteristic size of the diffracting body, the far-field back-scattered impulse response of the object can be seen to be proportional to the second derivative of the cross-sectional area function of the object in the transmitted-receiver direction with the time-space variable change. This identity is known as the Bojarski identity [6] in the context of electromagnetic scattering. Its inversion leads to the POFFIS (Physical Optics Far Field Inverse Scattering) identity, which is used in radar and underwater technology for identifying the shape of targets [7].

In ultrasonic non-destructive testing, since the same transducer is used for both the transmission and the reception of the waves, the signal recorded is the convolution between the back-scattered impulse response of the medium and the impulse response of the whole electro-acoustic system, which is defined as the back-scattered impulse response of a perfect normal flat reflector.

If the crack, which is assumed to be sufficiently open, is modeled in the form of a non-thin parallelepiped and if we consider only the terminal portion of the crack, the cross-section area function of this diffracting object is a Heaviside function and its second derivative is the first derivative of a Dirac distribution. The echographic response of the terminal portion of the crack will therefore be the first derivative of the impulse response of the electro-acoustic system, i.e. the first derivative of the response to a normal flat reflector.

The results obtained with this model show good agreement with our experimental data. It can be used to discriminate between the echoes back scattered by a crack and by an interface by performing spectral analysis: the transfer function between the two echoes must be a ramp function [8-9].

When the crack-tip and the interface coincide, however, the spatial (temporal) resolution, lost in the case of classical Fourier analysis, has to be preserved. It is therefore necessary to perform time-scale analysis using wavelet analysis methods, for example.

I. Far-field back-scattered impulse response of a scatterer

Let us take a convex obstacle V, which gives rise at the observation point N to a scattered field ψ_1. The scattered field (displacement) can be modeled by the following integral in terms of the density $\tilde{\mu}$ and the 3-D Green function g, which is the solution of the Dalembert homogenous equation, on the surface S (the boundary of V):

$$\psi_1(N, t) = -\iint_{t'S} g(N, t; P, t')\ \tilde{\mu}(P, t')\ dt'\ dP \tag{1}$$

Since $g(\vec{r}, t) = \frac{1}{4\pi r}\,\delta\!\left(t - \frac{\vec{r}}{c}\right)$ where $r = |NP|$ and c is the compressional wave velocity (acoustical assumptions only), we obtain the following expression for the total field ψ

$$\psi(N,t) = \psi_0(N,t) - \frac{1}{4\pi}\int_S \frac{1}{r}\,\tilde{\mu}(P, t - \frac{\vec{r}}{c})\,dP \qquad \textbf{(2)}$$

where $\psi_0(N, t)$ is the incident field at the point source A.

To introduce the Kirchhoff approximation for the field on S, we divide the surface S into the "lit" and "dark" side depending on the straight rays emitted by the point source A, which are incident on S^+ (Figure 1). We assume that the exact boundary condition stipulates that the total field must be zero on S^+. Under this condition, the density $\tilde{\mu}$ is given by:

$$\tilde{\mu}(P,t) = 2\,\frac{\partial\psi_0(P,t)}{\partial n'} \qquad \textbf{(3)}$$

and the diffracted field by:

$$\psi_d(\vec{x},t) = -\frac{2}{4\pi}\int_{S^+} \frac{1}{|\vec{x}-\vec{x}'|}\,\frac{\partial}{\partial n'}\psi_0(x', t - \frac{|\vec{x}-\vec{x}'|}{c})\,d^2\vec{x} \qquad \textbf{(4)}$$

where $\underline{\vec{x}'}$ is the distance from the origin O to point P on S^+, and $\underline{\vec{x}}$ is the distance to point N.

We now assume that the receiver and the transmitter are located far from the scatterer. Taking the origin O to be inside the obstacle, we then use the following asymptotic approximation:

$$|\vec{x}-\vec{x}'| \cong |\vec{x}| - \vec{n}\,.\,\vec{x} \qquad \textbf{(5)}$$

where $\vec{n} = \frac{\vec{x}}{|\vec{x}|}$ is the unit normal vector in the observed direction. The following result is obtained for the diffracted field:

$$\psi_d^\infty(\vec{x}, t) = -\frac{2}{4\pi|\vec{x}|}\left[\int_{S^+} \frac{\partial}{\partial n'}\psi_0(x', t' - \frac{\vec{n}.\vec{x}'}{c})\, d^2\vec{x}'\right]_{t' = t - |\vec{x}|/c} \quad \textbf{(6)}$$

The value of the incident field ψ_0 is the free-space solution of the wave equation:

$$\psi_0(\vec{x},t) = -\frac{f(M)}{4\pi|\vec{x}_0 - \vec{x}|}\,\delta\left(t - \frac{|\vec{x}_0 - \vec{x}|}{c}\right) \quad \textbf{(7)}$$

where $f(M)$ is a function describing the directivity of the source (typically a transducer with a directivity which is restricted to an axial cone, and no side lobes are take into account).

Introduce the asymptotic behavior:

$$|\vec{x}_0 - \vec{x}| = |\vec{x}_0| + \vec{n}_0 . \vec{x} \quad \textbf{(8)}$$

where $\vec{n}_0$ is the unit normal vector in the incident direction.

From these assumptions, we can also conclude that the diffracted field can be written:

$$\psi_d^\infty(\vec{x}, t) = -\frac{2\, f(M)}{4\pi|\vec{x}|\,|\vec{x}_0|}\left[\int_{S^+} \frac{\partial}{\partial n'}\delta(t' - \frac{(\vec{n} - \vec{n}_0).\vec{x}'}{c})\, d^2\vec{x}'\right]_{t'=t-(|\vec{x}|+|\vec{x}_0|)/c} \quad \textbf{(9)}$$

If a single transducer is used, we have $\vec{n} = -\vec{n}_0 = (1, 0, 0)$, and the measured field is the back-scattered field:

$$\psi_r^{\infty}(\vec{x}, t) = -\frac{f(M)}{|\vec{x}|^2} h_r^{\infty}\left(\vec{x}, t - \frac{2|\vec{x}|}{c}\right) \qquad \textbf{(10)}$$

where

$$h_r^{\infty}(\vec{x},\ t) = \frac{1}{8\pi^2} \int_{S^+} \frac{\partial}{\partial n'} \delta(t - \frac{2x'_i}{c})\, d^2\vec{x}' \qquad \textbf{(11)}$$

is the infinite back-scattered impulse response of the obstacle:

$$h_r^{\infty}(\vec{x}, t) = \frac{c}{8\pi^2} \int_{S^+} \delta'(x_i^{'} - \frac{c\,t}{2})\, d^2\vec{x}' \qquad \textbf{(12)}$$

We then decompose the volume of the obstacle into a "lit" part with a surface S^+ consisting of the wave front and the last section before the dark part, and we define the characteristic function of the "lit" part as follows:

$$\Gamma(\vec{x}) = \left| \begin{array}{l} = 1 \text{ if } \vec{x} \in V^+ \\ = 0 \text{ if not} \end{array} \right. \qquad \textbf{(13)}$$

and the cross-sectional area function of the scatterer (Figure 2):

$$A(\vec{x}) = \int_{V^+ \cap x_1^{\perp}} \Gamma(x)\, dx_2^{'} dx_3^{'} \qquad \textbf{(14)}$$

We then obtain:

$$h_r^{\infty}(\vec{x}, t) = \frac{c}{8\pi^2}\left[\frac{\partial^2}{\partial x^2} A_{V^+}(x)\right]_{x=ct/2} \qquad \textbf{(15)}$$

The far-field back-scattered impulse response of an obstacle is proportional to the second derivative of the cross-sectional area function, in the transmission/reception direction, after making the appropriate change in the time-space domain.

II. Diffraction of a crack tip

If we now introduce a non-thin crack modeled in the form of a stiff rectangular parallelepiped (Figure 3) with no internal propagation, the area function can be written:

$$A_{V^*}(x_1) = L_y L_z Y(x_1) \qquad \textbf{(16)}$$

where $Y(x_1)$ is the Heaviside function, L_y is the thickness of the crack, which is much smaller than the wavelength λ in the material, and L_z is the depth of the crack, which is much larger than the thickness. Since it has been established that:

$$\frac{\partial^2 Y(x)}{\partial x^2} = \frac{\partial \delta(x)}{\partial x} \qquad \textbf{(17)}$$

The far field back-scattered impulse response of the open tip of a non-thin crack is proportional to the first derivative of a Dirac distribution:

$$h_r^{\infty}(\vec{x}, t) = \frac{c}{8\pi^2} L_y L_z \frac{\partial}{\partial x} \delta\left(\frac{ct}{2}\right) \qquad \textbf{(18)}$$

In the frequency domain, the transfer function is:

$$H_r^\infty(k) = -\frac{ik}{2\pi} L_y L_z \tag{19}$$

where k is the wave number of the compressional wave.

III. Ultrasonic modeling

The data required here obviously has to be collected over a broad range of arbitrary directions. In our application, the scatterer is immersed in a water bath and the back-scattered echoes are measured at various incident angles. The ultrasonic mechanical device is therefore composed of a main symmetric arm, and since the transducer can be exactly positioned and oriented, both linear and sectorial scanning can be performed. Six stepping motors sequentially driven by a programmable translator-indexer device fitted with a power multiplexer are used to generate all the movements. The translator-indexer device and the power multiplexer are integrated into a control rack that also includes other remote control devices. The increments are multiples of 0.75 10^{-2} millimeter for translations and of 10^{-2} degree for rotations. The transducers are Panametric® transducers with a nominal frequency of 5 MHz. The Panametric® multiplexer excites the transducers and organizes the ultrasonic Radio-Frequency (RF-) signals received, which were composed of 256 samples and were digitized (8 bits, 20 MHz).

The back-scattered impulse response of the scatterer measured at angle ϕ can be written (Figure 4):

$$s(t) = \left(h_t \otimes h_m \otimes s_i\right)(t) \quad \textbf{(20)}$$

where $h_t(t)$ is the impulse response of the transducer including the whole electro-acoustic apparatus, which is assumed to be linear, $h_m(t)$ is the impulse response of the scatterer and $s_i(t)$ is the source, which is assumed to be a perfect electronic pulse $\delta(t)$.

$\otimes$ denotes the operation of "convolution".

If the scatterer is a perfect flat reflector, the back-scattered impulse response can be assumed to be a Dirac function (distribution):

$$h_m(t) = \delta(t) \quad \textbf{(21)}$$

we obtain:

$$s(t) = \left(h_t \otimes \delta \otimes \delta\right)(t) = h_t(t) \quad \textbf{(22)}$$

<u>First result</u>:

In the case of a perfect flat reflector, which in the present case was a rectified brass plate (Figure 5 and Figure 6), the RF-signal recorded in the back-scattered mode will be a "copy" of the impulse response of the transducer:

$$s(t) = h_t\left(\frac{ct}{2}\right) \tag{23}$$

If the scatterer is a crack tip, however, $h_m(t)$ is given by Eq. (18), and s(t):

$$s(t) = \left(h_t \otimes h_r^{\infty}\right)(t) = \frac{c}{4\pi} L_y L_z \left(h_t \otimes \frac{\partial}{\partial t}\delta\left(\frac{ct}{2}\right)\right) \tag{24}$$

<u>Second result</u>:

The signal diffracted by a non-thin crack tip is a "copy" of the first derivative (Figure 7) of the impulse response of the transducer:

$$s(t) = C\,\frac{\partial}{\partial t} h_t\left(\frac{ct}{2}\right) \tag{25}$$

where C is a constant.

IV. Results

The present study focuses on an industrial problem, namely the non-destructive testing of thick cracked stainless steel/steel coated plates measuring 10 cm in width and thickness, and 30 cm in length. The stainless steel coating was 1 cm in width and thickness, and 30 cm in length. The compressional wave velocity measured was 5866 m/s in the steel, and was 6330 m/s in the stainless steel. Figure 8 shows the enhancement procedure used, which was adapted for use with this ultrasonic method of assessment. A transducer is moved linearly along the interface between the water and the steel target (on the steel side), working from the intact part to the damaged part. In the case of protocol "*A*", the transducer forms an incident angle of 0° with respect to the normal vector of the water/steel interface. In the case of protocol "*B*", the transducer is tilted so that the incident beam reaching the interface between the two steels is directed at an angle of 60°, in keeping with Snell-Descartes laws. Since this angle is lower than the critical angle (68°) of the compressional waves, no shear waves propagation is taken into account in this case.

The interface between the two steels is plane and weakly contrasted. Any cracks occurring will develop near the interface, approximately perpendicularly to it, so that the crack tip echo and the specular echo generated by the interface will completely overlap when normal ultrasonic inspection procedures are used (protocol "*A*"). The problem is therefore not how to discriminate between crack and interface, but more subtly, how to discriminate between a cracked interface and a healthy interface. The ultrasonic power of the back-scattered echoes generated by the interface between the two steels and by the crack tip is weak enough to be able to assume that the interference introduced by the wave propagation is linear, and that the corresponding signal result from the sum of two echoes generated by the interface and the crack tip. The modeling problem can then be solved with a weak scattering solution, such as the physical optics approximation. Each signal is modeled separately. When dealing with the crack, the reference echo obtained on a brass-rectified plate was used for the derivation, and when dealing with the interface, we focused on the back-scattered echo originating from the interface between the two steels.

Figure 9 shows the experimental signal obtained from a back-scattered echo at the interface between the two steels in the intact region (protocol "*A*"). Figure 10 shows the experimental signal obtained from the back-scattered echoes originating from the crack tip (protocol "*B*"). In this case, there are no specular echoes because the incident beam is at an angle of 60°, and the signal recorded corresponds to the echo generated by the tip of the crack alone. Figure 11 shows the normalized comparison between the two signals.

Figure 12 shows the modulus of the spectra of the echoes generated by the interface between the two steels (solid line) and the crack tip (plus line). The division was made in the bandwidth of the transducer (point line) at –6 dB. Figure 13 gives the spectral law (solid line) and the polynomial approximation (point line), which takes the form of a ramp function with a slope equal to 0.98. It can be concluded that the spectral division varies proportionally with the wave number k. This is in agreement with Eqs. (19) and (25). The impulse response of a crack tip is a first-order derivative of the impulse response of the transmitter depending on the incident field.

In Figure 14 and Figure 15, the experimental back-scattered echo generated by the crack tip can be compared with the model (i.e. the first-order derivative of the impulse response of the transducer, Figure 7). The two signals are in good agreement, which confirms the theoretical assumption adopted and the modeling procedure presented here.

Lastly the echo back scattered by the cracked interface is compared with the model. Figure 16 shows the echo obtained at the cracked interface using protocol "*A*" in the damaged region. This signal results in theory from the separate contributions of the echoes generated by the interface and by the crack.

Figure 17 shows the back-scattered impulse response of a cracked interface predicted by the model. This prediction is the numerical sum of the impulse response of the transducer and its first order approximate derivative. Figure 18 shows the normalized comparison between the two time graphs.

To check the validity of our modeling procedure, we used a time-scale approach to the signals, namely the wavelet analysis method (see appendix VI). An analysis was performed between scales j = -4 (0.83 MHz) and 0 (13.3 MHz), and $2^5 - 1$ intermediate scales (γ) were used between each entire scale. The experimental signal (Figure 19) and the model (Figure 20) for the interface in the intact region (signals on the left) and for the cracked interface in the damaged region (signals on the right) were analyzed. The maximum coefficient was obtained for the model on the scale γ = -2.06 and for the experimental signal on the scale γ = -2.15. The behavior of the material on a scale corresponding to low frequencies, as well as the timing of events was very similar in both cases.

In both analyses, we noted the halving of the decompositions on scales $\gamma > -1.5$, 4.71 MHz, near the nominal frequency of the transducer, 4.3 MHz, which extended a little less towards scale 0 in the case of the model, however.

Two conclusions can be drawn from the results of the present study. First, the comparisons between the wavelet analysis of the experimental and simulated signals confirmed that there was good agreement between the predictions of the model and the experimental data, as regards the acoustical signature of the damaged/cracked interface and that of the intact interface. The modeling procedure presented here therefore provides a useful tool, since it is a simple method giving accurate results.

Secondly, the wavelet method can be used to discriminate between an echo resulting from an intact region and one resulting from a damaged region. The halving of the upper part of the wavelet decomposition showed the existence of two contributions, corresponding to the echoes originating from the interface and from the crack.

V. Conclusion

Here we address the problem of detecting and characterizing cracks in the field of non-destructive testing of materials, taking a signal processing approach. In the frequency domain, the ratio between the signal back scattered by an intact interface and that back scattered by the tip of an open crack was found to be proportional among the waves number k. Based on the physical optics approximation, it is established that the impulse response of scatterers of this kind is associated with the first-order derivative of a Dirac distribution. Adapting this response to the electro-acoustical context, the impulse response of a non-thin open crack can be said to be associated with the first-order derivative of the impulse response of the transducer, i.e. the incident wave. This result, which has often been assumed to be true in the literature, is proved here explicitly, adopting the hypotheses and the experimental conditions involved in real-life tests. The predictions of the model developed here and the results obtained by performing wavelet analysis on real and simulated signals show this theoretical law to be in particularly good agreement with the experimental data.

The present method based on a combination of modeling and wavelet analysis provides a useful time-frequency tool for processing the signals and analyzing the physical processes involved in the interactions between ultrasonic waves and materials. It could serve as the basis for developing industrial methods of non-destructive *in situ* crack testing.

VI. Appendix

For the wavelet algorithm, a redundant algorithm in scales, which was developed specifically for ultrasonic signal processing, was used [10]. It quickly calculates the wavelet coefficients of the signal on the time and (frequency) scale bands.

The time-frequency and time-scale analyses are often used when processing biological signals [11]. Wavelet decomposition of the signals is a transformation that depends on discrete parameters [12].

We associated the series of coefficients with a signal S as follows:

$$C_{j,k} = \int_{-\infty}^{\infty} S(t)\, \overline{\Psi}_{j,k}(t)\, dt = \int_{-\infty}^{\infty} S(t)\, \Psi_{j,k}(t)\, dt \qquad \textbf{(A.1)}$$

where *(j, k)* belongs to Z^2. When $j < 0$, we have dilation and when $j > 0$, we have compression. $C_{j,k}$ is the coefficient associated with S on scale j and with sample k.

The redundant algorithm in scales is an extension of the definitions published in studies on non-integer scales. In particular, we extended the definition of $C_{j,k}$, $j \in Z$ to $\gamma \in R$.

The wavelet Ψ was previously analyzed by S. Jaffard [13, 14]. Its spectral modulus reaches a maximum at 2/3 F_e, where F_e is the frequency rate of the signal (20 MHz in this study). It has a bandwidth equal to F_e and a –3dB bandwidth equal to ½ F_e. In the time domain, it is real and lower than 10^{-3} except for an interval $13U_t$ ($U_t = 1/Fe$) in length centered on ½ U_t , which is symmetric with respect to ½ U_t.

VII. References

1 Morse P.M. and Ingard K.U., "Theoretical acoustics", Mc Mraw-Hill Book company, New-York, 1968

2 De Vadder D., Dosso M., "Caractérisation ultrasonore des bords de fissure par traitement numérique du signal", Proc. 3rd European Conference on Non Destructive Testing, vol. 5, 1984 p. 362-374

3 Borovikov, V. A., Kinber, B. Y., "Geometrical Theory of Diffraction", IEE Electromagnetic Series, Stevenage, 1994, Vol. 37

4 Fradkin L. J., Kiselev A.P. , Krylova E., "The radiating near-field asymptotics of a normal time-harmonic circular ultrasonic transducer in an elastic half-space", J. Acoust. Soc. Am., **104**(3), 1998, p. 1178-1187

5 Keith G.M., Peake N., "High-wavenumber acoustic radiation from a thin-walled scarfed cylinder", J. of Sound and Vibration, 2002, p 147-160

6 Bojarski N., "A survey of the Physical Optics Inverse Scattering Identity", IEEE trans Antennas and Propagation, **30**, 5, 1982, p. 980-989

7 Li W., Liu G.R., Varadan V.K., "Fredholm integral equation for shape reconstruction of an underwater object using backscattered ramp response signatures" J. Acoust. Soc. Am. **116**, 3, 2004, p. 1436-1445

8 Varadan V.V., Tsao S.J., Varadan V.K., Tittmann B.R., "Image reconstruction of flaws using ramp response signatures", SPIE Int. Symp. on Pattern Recognition and Acoust. Im., **768**, 1987, p. 293-300

9 Varadan V.V., Tsao S.J., Varadan V.K., Tittmann B.R., "Image reconstruction of flaws using ramp response signatures", J. Wave-Mater. Interact., **10**, 1995, p. 67-78

10 Lasaygues P., Lefebvre J.P., Mensah S., "Deconvolution and Wavelet Analysis on Ultrasonic Reflexion Tomography", Topics On Non Destructive Evaluation Series B. Djordjevic and H. Dos Reis Series Eds, ANST III International Workshop - Advances in Signal Processing for NDE of Materials, in X. Maldague Ed **3;** 1998, p. 27–32

11 Akay M. "Time frequency and wavelets in biomedical signal processing", IEEE Press Series in Biomedical Engineering, in Metin Akay Series Editor ; 1997

12 Meyer Y., "Orthonormal wavelets", Proceeding Conf. Wavelets, Time-Frequency Method and Phase Space, Combs J.M. et al. Eds, Lectures notes on IPTI, Springer Verlag, 1989

13 Jaffard S., "Décompositions en ondelettes “Developments of Mathematics 1950-2000” (J-P. Pier Ed.) Birkhauser, 2000, p. 609-634 (in French)

14 Jaffard S., Meyer Y., and Ryan R., "Wavelets: Tools for Science and Technology", SIAM, 2001